A New Notion of Proportionality for Praticipatory Budgeting with Additive Utilities

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August 21, 2025

Abstract

Selecting representatives based on voter preferences is central in computational social choice, and participatory budgeting (PB) exemplifies this challenge by allocating limited funds to desired projects. We introduce Full Proportional Justified Representation (FPJR), a new proportionality axiom tailored to participatory budgeting with additive utilities. FPJR's cohesiveness criterion focuses on a voter's total utility rather than stringent per-project utility bounds, making it more natural in utility-based settings. We show that FPJR is incomparable to the previously studied Extended Justified Representation (EJR) and that the Method of Equal Shares (MES) achieves a 1/2 approximation to FPJR under 0/1 utilities. However, for general utilities, MES fails to guarantee FPJR or any constant approximation. These findings underscore the complexity of designing mechanisms that ensure robust proportionality in utility-based participatory budgeting and leave the question of whether such mechanisms exist open.

1 Introduction

Multiwinner elections, in which a committee of winners is selected based on voter preferences, have been extensively studied in the literature (see, e.g., Aziz et al. [2018a, 2017], Sánchez-Fernández et al. [2017], Peters and Skowron [2020], Lackner and Skowron [2023]). A prominent application of these frameworks is Participatory Budgeting (PB), where communities allocate limited public funds to projects based on residents' preferences Aziz and Lee [2021], Peters et al. [2021a,b], Aziz and Shah [2021], Brill et al. [2023], Aziz et al. [2018b]. PB has seen widespread adoption across many cities, and the variant where each voter provides a utility value for every project has recently drawn special interest (see, e.g., Peters et al. [2021a]).

A central concern in both multiwinner elections and participatory budgeting is proportional representation. In essence, proportionality axioms ensure that sufficiently large and cohesive groups of voters receive fair representation. Various definitions of "cohesiveness" and "representation" have been introduced, particularly in the context of approval-based voting: On the "cohesiveness" axis:

(C1) Two common axioms, Extended Justified Representation (EJR) and Proportional Justified Representation (PJR) consider a group cohesive if all its voters unanimously approve a proportionate number of candidates.

(C2) Other two common axioms, Full Justified Representation (FJR) and Full Proportional Justified Representation (FPJR) consider a group cohesive if each voter in the coalition approves at least a fixed fraction of a candidate subset proportional to the coalition's size.

On the "representation" axis:

(R1) PJR and FPJR require that the coalition's collective utility from the winning set exceeds that of any proportionally sized alternative.

(R2) EJR and FJR require that for any proportionally sized alternative, at least one member of the coalition derives less utility from it than from the winning set.

Recently, [Peters et al., 2021a] adapted these notions to the utility-based PB setting. They showed that natural extensions of EJR and FJR lead to NP-hard computational problems, primarily because these scenarios reduce to knapsack-like problems. To address this complexity, they proposed relaxed variants, such as EJR-1, where the axiom can be met by hypothetically adding one more project. They proved that the Method of Equal Shares (MES) achieves EJR-1. They also demonstrated that outcomes satisfying EJR and FJR exist, but remain computationally elusive.

Against this background, our monograph addresses two main questions. First, we emphasize that in the participatory budgeting with utilities setup, "a fraction of approval" perspective (C2) seems more natural as it does not require individual utility lower-bounds on projects. This observation prompts us to ask: can we design efficient mechanisms that satisfy axioms with (C2) type cohesiveness characteristics, i.e., FJR or FPJR? Recent result of [Kalayci et al., 2024] shows that EJR and FPJR are incomparable, which further motivates to investigate FRJR axiom. Second, although exact compliance is difficult, approximation techniques for knapsack-like problems are well-understood. This suggests exploring whether JR-type axioms can be approximated rather than relying solely on relaxations like EJR-1.

In this work, we take initial steps to answer these questions. First, we formally introduce the FPJR axiom, which extends the family of proportional representation guarantees to account for collective utility of the coalition over selected projects. Second, we define approximate variants of EJR, FJR, PJR, and FPJR. In particular, we introduce γ-FPJR, under which the utility of an outcome for a given coalition is measured by aggregating, over all selected projects, the highest voter utility that any coalition member assigns to that project. The γ-FPJR axiom requires that this total outcome utility for every cohesive coalition be at least γ times the minimal total utility benchmark, where the benchmark is defined as the smallest among the utilities that individual voters in the coalition can achieve from any proportionate selection of projects (i.e., a set of projects whose total budget is at most the coalition's share of the overall budget).

Next, we apply this framework to the Method of Equal Shares (MES), a prominent rule for allocating resources. Under 0/1 utilities, i.e., approval utilities, we show that MES achieves a 1/2-FPJR approximation guarantee, demonstrating that it provides a substantial fraction of the proportional utility for every cohesive coalition. However, we also prove a fundamental limitation: MES cannot guarantee β-FPJR for any $\beta > 0$. This result highlights the inherent computational challenges in fully realizing these approximate proportionality guarantees.

These findings open several avenues for future research. Can we identify a rule that approximately satisfies FPJR? Under approval utilities, can MES achieve a better approximation of FPJR, or even achieve exactly FPJR rather than just an approximation? Furthermore, analogous to EJR-1, one could define FPJR-1 and investigate whether MES meets this relaxed requirement. Since EJR and FPJR are incomparable axioms, designing mechanisms that ensure FPJR remains a challenging and significant problem for future work.

2 Preliminaries

In this section, we formalize the participatory budgeting (PB) model and introduce relevant concepts and notation.

Definition 2.1 (Election)**.** *For each positive integer t, let $[t] = \{1, \ldots, t\}$. An* ***election*** *is defined by:*

- *A set of* ***voters*** *$N = [n]$ and a set of* ***candidates*** *$C = \{c_1, \ldots, c_m\}$.*
- *A* ***budget*** *$b \in \mathbb{R}_{\geq 0}$.*
- *A* ***cost function*** $\mathrm{cost} : C \to \mathbb{R}_{\geq 0}$ *assigning a cost to each candidate. For $T \subseteq C$, we define* $\mathrm{cost}(T) = \sum_{c \in T} \mathrm{cost}(c)$*.*
- *For each voter $i \in N$, an* ***utility function*** *$u_i : C \to \mathbb{R}_{\geq 0}$, assigning a nonnegative utility to each candidate for voter i. For $T \subseteq C$, define $u_i(T) = \sum_{c \in T} u_i(c)$, and for $S \subseteq N$, $u_S(T) = \sum_{i \in S} u_i(T)$.*

We assume that $u_N(c) = \sum_{i \in N} u_i(c) > 0$ for every $c \in C$, ensuring that each candidate provides positive utility to at least one voter.

Definition 2.2 (Feasible Set, Outcome, and Rule)**.** *Given an election $E = (N, C, b, \mathrm{cost}, (u_i)_{i \in N})$:*

- *A subset $T \subseteq C$ is* ***feasible*** *if* $\mathrm{cost}(T) < b$*.*
- *A* ***outcome*** *of the election is a feasible subset $W \subseteq C$.*
- *A* ***rule*** *is a function $\mathcal{R}$ mapping each election E to a feasible outcome $W = \mathcal{R}(E)$, called the* ***winning committee****.*

The model described above is referred to as the ***general PB model****.*

Definition 2.3 (Special Cases)**.** *We consider two special cases of the general PB model:*

- *A* ***committee selection*** *is an election where b is an integer and* $\mathrm{cost}(c) = 1$ *for every $c \in C$. In this case, an outcome is called a* ***committee****, and the election is under the* ***unit-cost assumption****.*
- *An election is with* ***approval utilities*** *if $u_i(c) \in \{0, 1\}$ for all $i \in N$ and $c \in C$. Define $A_i := \{c \in C : u_i(c) = 1\}$ and say that i* ***approves*** *c if $c \in A_i$. We say that $c \in W$* ***represents*** *i if $c \in A_i$.*

An ***approval-based committee selection*** *is a committee selection with approval utilities.*

These special cases have motivated many ideas in the PB model.

A central goal in participatory budgeting is ensuring that sufficiently large minority groups have a voice in the outcome. In computational social choice, this concept is captured by *proportionality*, which guarantees that large, cohesive groups with shared interests receive a proportionate share of the budget.

To formalize this, we first introduce two notions of group cohesiveness:

Definition 2.4 ((α, T)-Cohesiveness)**.** *Let $\alpha : C \to [0,1]$ be a function, and let $T \subseteq C$ be a subset of candidates. A group of voters $S \subseteq N$ is **(α, T)-cohesive** if*

$$\frac{|S|}{|N|} \geq \frac{\text{cost}(T)}{b} \quad \textit{and} \quad u_i(c) \geq \alpha(c) \textit{ for all } i \in S \textit{ and } c \in T.$$

Definition 2.5 (Weak (β, T)-Cohesiveness)**.** *Let $\beta \in \mathbb{R}_{\geq 0}$ and $T \subseteq C$. A group of voters $S \subseteq N$ is **weakly (β, T)-cohesive** if*

$$\frac{|S|}{|N|} \geq \frac{\text{cost}(T)}{b} \quad \textit{and} \quad u_i(T) \geq \beta \textit{ for all } i \in S.$$

*We sometimes refer to T as the **witness set** of S.*

The concept of proportional representation is captured by axioms such as Extended Justified Representation (EJR) and Full Justified Representation (FJR):

Definition 2.6 (Extended Justified Representation (EJR))**.** *A rule $\mathcal{R}$ satisfies **EJR** if, for every election E and every (α, T)-cohesive group S, there exists some $i \in S$ such that*

$$u_i(\mathcal{R}(E)) \geq \sum_{c \in T} \alpha(c).$$

Definition 2.7 (Full Justified Representation (FJR))**.** *A rule $\mathcal{R}$ satisfies **FJR** if, for every election E and every weakly (β, T)-cohesive group S, there exists some $i \in S$ such that*

$$u_i(\mathcal{R}(E)) \geq \beta.$$

Note that every (α, T)-cohesive group is also weakly $(\sum_{c \in T} \alpha(c), T)$-cohesive, but not necessarily the other way around. Thus, EJR is a strictly weaker requirement than FJR.

3 Approximated Proportionality and FPJR

It is known that no aggregation rule running in strongly polynomial time can satisfy FJR or EJR in the general PB model, unless $\mathbf{P} = \mathbf{NP}$. This follows from the fact that, when restricted to a single voter, determining whether FJR or EJR can be satisfied reduces to solving the **Knapsack Problem** optimally. Hence, we do not expect to find polynomial-time rules that satisfy these axioms.

Any constant approximation of the Knapsack problem can be found in polynomial time, so a natural next step is to consider approximate versions of these proportionality axioms. To that end, the following definitions are introduced:

Definition 3.1 (γ-EJR)**.** *Let $0 < \gamma \leq 1$. A rule $\mathcal{R}$ satisfies **γ-EJR** if, for every election E and every (α, T)-cohesive group S, there exists $i \in S$ such that*

$$u_i(\mathcal{R}(E)) \geq \gamma \cdot \sum_{c \in T} \alpha(c).$$

Similarly, we define γ-FJR:

Definition 3.2 (γ-FJR)**.** *Let $0 < \gamma \leq 1$. A rule $\mathcal{R}$ satisfies **γ-FJR** if, for every election E and every weakly (β, T)-cohesive group S, there exists $i \in S$ such that*

$$u_i(\mathcal{R}(E)) \geq \gamma \cdot \beta.$$

Inspired by the concept of *Full Proportional Justified Representation* (FPJR) Kalayci et al. [2024], originally defined for approval-based committee selections, we now introduce a generalized version of FPJR for the PB setting:

Definition 3.3 (Full Proportional Justified Representation (FPJR))**.** *A rule $\mathcal{R}$ satisfies **FPJR** if, for every election E and every weakly (β, T)-cohesive group S, the winning set $W = \mathcal{R}(E)$ satisfies*

$$\sum_{c \in W} \max_{v \in S} u_v(c) \geq \beta.$$

This definition generalizes the notion of FPJR to the PB model and coincides with the original FPJR definition when restricted to approval-based committee selections. By definition, FPJR is a weaker requirement than FJR; this inclusion is strict, as will be discussed.

It is not evident whether there is a direct relationship between EJR and FPJR. In fact, they are incomparable, even in approval-based committee selections. Consequently, FJR is strictly stronger than FPJR.

Example 3.4 (EJR but not FPJR)**.** *Consider an approval-based committee selection with 15 candidates and $n = 6$ voters $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, with approval sets:*

$$\begin{aligned}
A_{v_1} &= \{c_1, c_2, c_3, c_4\}, \\
A_{v_2} &= \{c_1, c_2, c_3, c_5\}, \\
A_{v_3} &= \{c_1, c_2, c_3, c_6\}, \\
A_{v_4} &= \{c_7, c_8, c_9\}, \\
A_{v_5} &= \{c_{10}, c_{11}, c_{12}\}, \\
A_{v_6} &= \{c_{13}, c_{14}, c_{15}\}.
\end{aligned}$$

Let the committee size be $k = 12$. The committee

$$W = \{c_1, c_2, c_3, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13}, c_{14}, c_{15}\}$$

satisfies EJR. However, consider the 4-cohesive coalition $S = \{v_1, v_2, v_3\}$ with a witness set $T = \{c_1, c_2, c_3, c_4, c_5, c_6\}$. This group collectively approves only three candidates in W, violating FPJR.

Example 3.5 (FPJR but not EJR)**.** *Let $k = n = 2$. Suppose there are two voters v_1, v_2 with:*

$$A_{v_1} = \{c_1, c_2\}, \quad A_{v_2} = \{c_1, c_3\}.$$

The committee $W = \{c_2, c_3\}$ satisfies FPJR but not EJR, as it provides only one representative per voter for the 2-cohesive group $\{v_1, v_2\}$ with witness $T = \{c_1\}$.

As with EJR and FJR, there is no known rule computable in strongly polynomial time that satisfies FPJR in the general PB model, unless $\mathbf{P} = \mathbf{NP}$. Thus, attention naturally turns to approximation:

Definition 3.6 (γ-FPJR)**.** *Let $0 < \gamma \leq 1$. A rule $\mathcal{R}$ satisfies **γ-FPJR** if, for every election E and every weakly (β, T)-cohesive group S, the winning set $W = \mathcal{R}(E)$ satisfies*

$$\sum_{c \in W} \max_{v \in S} u_v(c) \geq \gamma \cdot \beta.$$

4 Method of Equal Shares and its Proportionality

In this section, we present a variant of the well-known Method of Equal Shares (MES) rule, originally developed for participatory budgeting with utilities. We then analyze its proportionality properties.

4.1 Method of Equal Shares

We begin by formally stating the MES rule, as introduced by [Peters et al., 2021a].

Consider an election instance $(C, \text{cost}, V, \mathcal{U})$ with a total budget B. Let $N = |V|$, where V is the set of voters. The MES rule initially assigns each voter $i \in V$ a personal budget $b_i = B/N$. It then iteratively selects the project that is "most affordable" according to a fairness criterion. In particular, MES chooses a project with minimum "cost per utility" ratio, where voters with higher utility for a project pay more, ensuring fairness. This idea is captured by the notion of ρ-affordability.

Definition 4.1 (ρ-affordable)**.** *For $\rho \geq 0$, a candidate $c \in C \setminus W$ is called **ρ-affordable** if*

$$\sum_{i \in V} \min(b_i, u_i(c) \cdot \rho) = \text{cost}(c).$$

Informally, c is ρ-affordable if it is possible to pay for c by charging each voter i at most $u_i(c) \cdot \rho$ from their remaining budget b_i.

The MES algorithm is given as follows.

Algorithm 1 Method of Equal Shares (MES)

Input: Election $(C, \text{cost}, V, \mathcal{U})$, Budget B.
Output: Set of selected projects W

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Let N = |V|.
Assign b_i ← B/N for all i ∈ V.
W ← ∅.
while there exists a ρ-affordable candidate for some ρ ≥ 0 do
    Let c be a ρ-affordable candidate with the minimum such ρ.
    W ← W ∪ {c}.
    for all i ∈ V do
        b_i ← b_i − min(b_i, u_i(c) · ρ).
Return: W.
```

4.2 Results

In this section, we aim to demonstrate proportionality characteristics of the method of equal share rule in terms of FPJR axiom. We first show that MES can not satisfy any γ-FPJR for any constant $\gamma > 0$.

Theorem 4.2. *For any constant $\gamma > 0$, MES violates γ-FPJR.*

Proof. For any fixed $\gamma > 0$, let M be an integer larger than $1/\gamma$. Let $V = \{v_1, \ldots v_{2M+1}\}$ be $2M+1$ voters, $C = \{c_1, c_2\}$ be two projects and the budget $B = 2M + 1$. Define the cost of both projects as $M + 1$ which means only one of them can be selected. Next, we define utilities as follows:

- Voters $\{v_1, v_{M+1}\}$ have utility 1 for project c_1 and utility $\frac{2}{2M+1}$ for project c_2.
- Voters $\{v_{M+2}, \ldots v_{2M+1}\}$ have utility 0 for project c_1 and utility 10 for project c_2.

Notice that project c_1 is 1 affordable but project c_2 is $\frac{\text{cost}(d)-M}{(M+1)\cdot\frac{2}{2M+1}} = \frac{2M+1}{2M+2} < 1$ affordable. Therefore, MES chooses project c_2.

Now, consider the coalition $S = \{v_1, \ldots v_{M+1}\}$ and observe that S is $(1, \{c_1\})$-cohesive. However, $\max_{i \in S} u_i(c_2) = \frac{2}{2M+1} < \frac{1}{M}$, and so the solution can not satisfy $\frac{1}{M}$-FPJR. □

Next, we show a positive result: MES satisfies 1/2-FPJR when preferences are approvals, i.e., $u_i(c) \in \{0, 1\}$ for all $i \in V$ and $c \in C$.

Theorem 4.3. *If $u_i(c) \in \{0,1\}$ for all $i \in V$ and $c \in C$, then MES satisfies 1/2-FPJR.*

To prove this theorem, we first establish a useful lemma.

Lemma 4.4. *Consider an election with approval utilities. Suppose that, at some step of MES, there is a coalition $S \subseteq V$ and a set $T \subseteq C$ such that:*

$$\sum_{i \in S} b_i \geq \text{cost}(T), \quad \textit{and} \quad \min_{i \in S} u_i(T) \geq \beta.$$

Then, there exists a candidae $c \in T$ such that $\sum_{\substack{i \in S \\ u_i(c)=1}} \min(b_i, 1/\beta) \geq \text{cost}(c)$. This also implies that c is $1/\beta$-affordable.

Proof. Suppose, for contradiction, that no candidate in T is $1/\beta$-affordable. Consider:

$$\begin{aligned}
\sum_{i \in S} b_i &\leq \beta \cdot \left(\sum_{\substack{i \in S \\ b_i < 1/\beta}} b_i + \sum_{\substack{i \in S \\ b_i \geq 1/\beta}} \frac{1}{\beta} \right) \\
&= \beta \cdot \sum_{i \in S} \min(b_i, 1/\beta) \\
&\leq \sum_{c \in T} \sum_{\substack{i \in S \\ u_i(c)=1}} \min(b_i, 1/\beta)
\end{aligned}$$

Since each voter in S approves all candidates in T, and none is $1/\beta$-affordable by assumption, we derive:

$$\sum_{c \in T} \sum_{\substack{i \in S \\ u_i(c)=1}} \min(b_i, 1/\beta) < \sum_{c \in T} \text{cost}(c) = \text{cost}(T),$$

contradicting the assumption $\sum_{i \in S} b_i \geq \text{cost}(T)$.

Thus, $\sum_{\substack{i \in S \\ u_i(c)=1}} \min(b_i, 1/\beta) \geq \text{cost}(c)$ for some $c \in T$. □

Proof of Theorem 4.3. Consider a weakly (β, T)-cohesive coalition S. By Lemma 4.4, there exists $c_1 \in T$ that is $1/\beta$-affordable. Selecting c_1 and deducting its cost from the budgets of S's voters still leaves a situation where $T \setminus \{c_1\}$ is affordable by S and the minimum utility of S on $T \setminus \{c_1\}$ is $\beta - 1$. Repeating this argument β times, we identify candidates $c_1, c_2, \ldots, c_\beta \in T$ where c_j is $1/(\beta - j + 1)$-affordable.

Let $W_S = \{c \in W : \max_{i \in S} u_i(c) = 1\}$ be the candidates chosen by MES that are approved by at least one member of S.

Claim 4.5. *For each $1 \leq j \leq \lceil \beta/2 \rceil$, the j-th candidate chosen by MES that is approved by S is $1/(\beta - j + 1)$-affordable.*

Proof of Claim. We prove this by induction on j.

Base Case: The first project chosen by MES is necessarily 1-affordable since every voter starts with budget 1. Since c_1 is $1/\beta$-affordable, if MES chooses some $w_1 \in W_S$ first, then w_1 must be at most $1/\beta$-affordable (otherwise it would not be chosen before c_1).

Inductive Step: Assume the statement holds for all $j \leq i$. After the first i projects that are $1/(\beta - j + 1)$-affordable have been chosen, each voter still has at least $1 - \sum_{j=1}^{i} 1/(\beta - j + 1)$ budget remaining. Since $\sum_{j=1}^{\lfloor \beta/2 \rfloor} 1/(\beta - j + 1) \leq 1/(\lfloor \beta/2 \rfloor)$, each voter still retains at least $1/\lceil \beta/2 \rceil$ of their budget. Thus, the not-chosen candidates among $\{c_1, \ldots, c_{i+1}\}$ remain feasible. Since each is $1/(\beta - i)$-affordable, the next chosen candidate from W_S must be at most $1/(\beta - i)$-affordable (even if only the voters in S pay, as the lemma shows).

This completes the induction. □

Suppose, for contradiction, that $|W_S| < \beta/2$. Then the total budget remaining after these $|W_S|$ selections is still at least $1/\lfloor \beta/2 \rfloor$. Hence, the candidates $c_1, \ldots, c_{\lfloor \beta/2 \rfloor}$ (all $1/\lfloor \beta/2 \rfloor$-affordable) could still be afforded solely by coalition S. This implies that MES could include more projects, contradicting the maximality of the chosen set W. Therefore, $|W_S| \geq \beta/2$, and MES satisfies 1/2-FPJR under approval utilities. □

5 Conclusion and Open questions

We have introduced a new proportionality axiom for utility-based participatory budgeting (FPJR) and explored its approximate variants. In particular, we showed that while MES cannot meet γ-FPJR for any positive constant γ in general, it does achieve a 1/2-FPJR approximation when utilities are restricted to {0,1}. This result provides a positive step forward for achieving meaningful proportionality guarantees in more tractable settings.

These findings, however, open several avenues for future research. A key question is whether it is possible to design an efficient mechanism that satisfies γ-FPJR for arbitrary utility functions and any constant $\gamma > 0$. Additionally, we ask if MES can achieve exact FPJR under 0-1 utilities, narrowing the gap between approximated and exact solutions. Beyond FPJR, one can consider relaxed notions such as γ-EJR or γ-FJR, prompting the question of whether efficient rules exist that meet these criteria. Finally, the concept of FPJR-1 (FPJR up to one additional project) invites further inquiry into whether there exist efficient mechanisms that can achieve this relaxed form of proportionality.

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